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ANNUALISED RETURNS AND CONTRIBUTIONS

FLAMETREE TECHNOLOGIES PTY LTD

Overview

This paper describes how returns and contributions in FIA are annualized.

Annualized total return is the geometric average of return earned by an investment each year over a given time period. For periods of less than a year, the annualized return is given by extrapolating the return seen over the period to the entire interval. In other words,

$$R = (1 + r)^{1/t} - 1 \tag{1}$$

where t is the fraction of a year over which r is measured. For instance, if a security has a return r of 1% over a period of 30 days, then its annualized return R is

$$R = (1 + 0.01)^{\frac{365}{30}} - 1 = 12.8695\%$$

Similarly, if the security has a compounded return of 20% over two years, then its annualized return is

$$R = (1 + 0.2)^{0.5} - 1 = 9.5445\%$$

Annualization and smoothing

For returns over more than one interval, FIA applies various types of smoothing to ensure that returns and contributions combine in a self-consistent manner. Smoothing is typically

- arithmetic: in which case all returns and contributions sum to the aggregated total;
- geometric: where all returns and contributions compound geometrically to the aggregatd totals

When applying annualization to FIA's output, we want to ensure that the annualized data also compounds in the same way as the interval data.

Geometrically smoothed data

For geometrically smoothed data, this is simply a question of applying equation (1) to all returns and contributions. In other words, each return r and contribution c is transformed as follows:

$$r \leftarrow (1+r)^{1/t} - 1$$

$$c \leftarrow (1+c)^{1/t} - 1$$

where t is the width of the interval, as a fraction of a year. Annualised returns and contributions will then compound geometrically in the same way as the original data.

Arithmetically smoothed data

Annualizing arithmetically smoothed data is a little more complex. At first glance, it might look as if each smoothed return and contribution should simply be multiplied by a factor $1/t$, which ensures that returns and contributions remain additive. However, the annualised total will be incorrect, because

$$R/t \neq (1+R)^{1/t} - 1$$

To correct this, calculate the annualized arithmetic and geometric returns R_a and R_g for r, and apply as follows:

$$r \leftarrow \frac{r \times R_g}{t \times R_a} \tag{2}$$

$$c \leftarrow \frac{c \times R_g}{t \times R_a} \tag{3}$$

where

$$R_a = r/t$$

$$R_g = (1+r)^{1/t} - 1$$

Example

Consider a security which shows a return of 1% over 30 days. In this case, $R_a = 0.12167$ and $R_g = 0.12869$. The annualised, compounded arithmetic return is then given by

$$R = \frac{0.01 \times 0.12869 \times 12.16667}{0.12167} = 12.8695\%$$

which is the geometrically compounded result.