



Price, YTM and discount yields in FIA

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INTRODUCTION

This paper describes the dependencies between supplied price and yields for securities in FIA, and where one set of data can be used to override another.

BACKGROUND

All fixed income securities modelled within FIA have the following quantities:

- a price. Usually, this is reported as the dirty price, which include accrued interest.
- a yield to maturity
- an associated yield curve

A typical security is priced by discounting each of its cash flows using the appropriate vector of yields from the yield curve, and then adding these discounted cash flows together to arrive at a dirty price.

The yield to maturity (YTM) of the security is the scalar-valued yield that can be used to discount each cash flow to give the dirty price.

In principle, a security need only have its price *or* its YTM supplied. From this, its corresponding YTM or price can be calculated.

The price of a bond p , and its yield to maturity y , are related by the expression (1)

$$p = \sum_i \frac{CF_i}{(1 + y_i)^{T_i}} = \sum_i \frac{CF_i}{(1 + y)^{T_i}}$$

where the sum is over the security's i cash flows: CF_i is the cash flow size, y_i the discount yield, and T_i the tenor (in years) of cash flow i . Other types of security are treated in a similar way.

MOTIVATION

Why should it be necessary to override data in FIA? The following case shows one example.

EXAMPLE

Suppose FIA is run on a corporate bond that trades at a 100 basis point spread to a similar Treasury security, but the only curve available is the risk-free curve.



FIA will price the security using the risk-free curve. As a result, its price will be over-estimated and its yield to maturity will be underestimated, leading to a lower carry return.

The security's return due to changes in the risk-free curve will not be affected, but the credit return (or residual) will be overestimated to compensate for the lower carry return.

One way around this issue is to supply an appropriate yield curve for the corporate bond. However, if for any reason the bond trades at a premium or a discount to its peers, then it will still not be priced correctly.

In this case, the best way forward is to supply a market YTM for the bond, in addition to the base data described above. This guarantees that the carry return is accurate. It also allows other measures of return to be measured accurately, such as the corporate bond's return from risk-free curve movements, and its spread return.

DATA FLOWS

FIA allows calculations to start from four different points, depending on what information is available.

CASE 1: YTM NOT SUPPLIED, P NOT SUPPLIED

The security's price is calculated using the yield curve, then its YTM is calculated from the calculated price, solving equation (1) numerically.

CASE 2: YTM SUPPLIED, P NOT SUPPLIED

The security is priced using the YTM rather than the yield curve, by passing it a flat curve with all its yields equal to the YTM. The supplied YTM is also used for all subsequent calculations: to calculate its carry return, and as a basis for risk analytics.

CASE 3: YTM NOT SUPPLIED, P SUPPLIED

- If the **ImpliedAnalytics** switch is on, YTM is calculated from the supplied price.
- If the **ImpliedAnalytics** switch is off, the supplied price is ignored and the security is priced from its yield curve as in case 1.

CASE 4: YTM SUPPLIED, P SUPPLIED

The security price is calculated from the YTM, and any supplied value of P is overridden. The calculation then proceeds as in case 2.

DEPENDENT ANALYTICS

Once a price and a YTM are available, FIA calculates some additional quantities:



MODIFIED DURATION

Irrespective of how the security is priced, FIA calculates modified duration by assuming a flat yield curve with a level equal to that of the security's YTM.

The MD is then given by the approximation

$$MD = -\frac{P(y + \delta) - P(y - \delta)}{2\delta P(y)}$$

where $P(y)$ is the price of the security with yield to maturity y .

CONVEXITY

Convexity C is given by the approximation

$$C = \frac{P(y + \delta) - 2P(y) + P(y - \delta)}{P(y)\delta^2}$$

where the yield curve is approximated using the same approach as for modified duration.

NOTES

YTM OVERRIDE

Cases 1 and 2 imply that a supplied YTM will always be used if available. This behaviour is hard-coded and cannot be overridden, irrespective of the setting of the **ImpliedAnalytics** switch.

A yield curve is only used in case 1, as in all other cases enough information is provided to provide a YTM and price.

IMPACT ON ATTRIBUTION CALCULATION

Does the use of a supplied YTM for pricing, rather than a full yield curve, have any impact on attribution?

The answer is, 'not very much'. Using YTM as a proxy for a full pricing curve means that some of the interim prices used to calculate, for instance, non-parallel shifts will be slightly less accurate than would be the case if the full curve were used. On the other hand, carry return will always be accurate, and the overall impact will be negligible unless the yield curve shows particularly large changes.