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EQUITY ATTRIBUTION MODELS IN FIA

FLAMETREE TECHNOLOGIES PTY LTD

Requirement

This paper provides a concise account of the equity attribution models available in FIA 2.4.2 upwards. The models may be classified as

- standard Brinson
- Brinson with normalised stock selection
- Bottom-up

The paper gives describes interaction and price effect, if applicable.

Notation

In the following presentation, lower-case variables refer to the portfolio, and upper-case variables to the benchmark.

At the security level, r_i denotes the return of security i in a portfolio, while R_i is the return of the same security in a benchmark. We do not necessarily assume that these quantities are the same; they may come from different sources, or the portfolio return may include transactions.

Similarly, w_i is the weight of security i in a portfolio, while W_i is the return of the same security in a benchmark.

In the same way, r_S and R_S are the returns of sector S in portfolio and benchmark, and w_S and W_S are the weights of the sectors. These are calculated as

$$w_S = \sum_{i \in S} w_i \tag{1}$$

$$W_S = \sum_{i \in S} W_i \tag{2}$$

$$r_S = \frac{\sum\limits_{i \in S} w_i r_i}{\sum\limits_{i \in S} w_i} \tag{3}$$

$$R_S = \frac{\sum_{i \in S} W_i R_i}{\sum_{i \in S} W_i} \tag{4}$$

Lastly, r and R (without subscripts) denote the return of portfolio and benchmark, respectively, given by

$$r = \sum_{i \in P} w_i r_i \tag{5}$$

$$R = \sum_{i \in B} W_i R_i \tag{6}$$

Brinson attribution

Consider a portfolio where the manager makes a single asset allocation decision in terms of allocations to sectors. The active return of the portfolio against its benchmark is given by $r - R$.

Asset allocation returns r_S^{AA} are given by

$$r_S^{AA} = (w_S - W_S) \times (R_S - R) \tag{7}$$

Note that asset allocation return is only described at the sector level.

Stock selection returns r_i^{SS} are given by

$$r_i^{SS} = W_i \times (r_S - R_S) \tag{8}$$

Interaction returns r_i^I are given by

$$r_i^I = (w_i - W_i) \times (r_S - R_S) \tag{9}$$

Lastly, pricing returns r_i^P are given by

$$r_i^P = w_i \times (r_i - R_i) \tag{10}$$

The sum of the four sources of return equals the overall active return of the portfolio against its benchmark:

$$r_S^{AA} + r_S^{SS} + r_S^I + r_S^P = r - R \tag{11}$$

Notes

- FIA supports multiple levels of asset allocation decision. In this case, there are as many asset allocation terms as there are sectors used.
- The presence of an interaction term is a shortcoming in the Brinson model. It is often combined with stock selection.
- In the Brinson model, asset allocation is only shown at the sector level, while all other sources of return are shown at the security level.
- In the case that $r_i = R_i$, pricing return is zero.

Normalised stock selection

Using the same notation as above, the normalised stock selection approach calculates stock selection as follows:

$$\widetilde{w}_i = w_i / w_S \tag{12}$$

$$\widetilde{W}_i = W_i / W_S \tag{13}$$

Stock selection return is given by

$$r_i^{SS} = w_S^P (\widetilde{w}_i - \widetilde{W}_i)(R_i - R_S) \tag{14}$$

Notes

- In this model, returns due to asset allocation and price are the same as for the Brinson approach.
- Interaction return does not arise using this model.

Bottom-up attribution

Here, asset allocation return is given by

$$r_S^{AA} = (w_i - W_i) \times (R_i - R) \tag{15}$$

and stock selection by

$$r_i^{SS} = w_i \times (r_i - R_i) \tag{16}$$

Notes

- Bottom-up attribution is the only model in which it makes sense to show asset allocation returns at the security level.
- Stock selection in the bottom-up model is the same as price return in the previous two models.
- Interaction return does not arise using this model.

Worked examples

The reader is referred to the accompanying spreadsheet 'Equity attribution using Brinson, normalised and bottom-up approaches' for a worked example of all returns shown in this paper.

References

'Equity attribution using Brinson, normalised and bottom-up approaches', Excel spreadsheet, Flametree, dated 15th October 2021