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EQUITY ATTRIBUTION MODELS IN FIA

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Requirement

This paper provides a concise account of the equity attribution models available in FIA 2.4.2 upwards. The models may be classified as

- standard Brinson
- Brinson with normalised stock selection
- Bottom-up

The paper gives describes interaction and price effect, if applicable.

Notation

In the following presentation, lower-case variables refer to the portfolio, and upper-case variables to the benchmark.

At the security level, r_i denotes the return of security i in a portfolio, while R_i is the return of the same security in a benchmark. We do not necessarily assume that these quantities are the same; they may come from different sources, or the portfolio return may include transactions.

Similarly, w_i is the weight of security i in a portfolio, while W_i is the return of the same security in a benchmark.

In the same way, r_S and R_S are the returns of sector S in portfolio and benchmark, and w_S and W_S are the weights of the sectors. These are calculated as

$$w_S = \sum_{i \in S} w_i \quad (1)$$

$$W_S = \sum_{i \in S} W_i \quad (2)$$

$$r_S = \frac{\sum_{i \in S} w_i r_i}{\sum_{i \in S} w_i} \quad (3)$$

$$R_S = \frac{\sum_{i \in S} W_i R_i}{\sum_{i \in S} W_i} \quad (4)$$

Lastly, r and R (without subscripts) denote the return of portfolio and benchmark, respectively, given by

$$r = \sum_{i \in P} w_i r_i \quad (5)$$

$$R = \sum_{i \in B} W_i R_i \quad (6)$$

Brinson attribution

Consider a portfolio where the manager makes a single asset allocation decision in terms of allocations to sectors. The active return of the portfolio against its benchmark is given by $r - R$.

Asset allocation returns r_S^{AA} are given by

$$r_S^{AA} = (w_S - W_S) \times (R_S - R) \quad (7)$$

Note that asset allocation return is only described at the sector level.

Stock selection returns r_i^{SS} are given by

$$r_i^{SS} = W_i \times (r_S - R_S) \quad (8)$$

Interaction returns r_i^I are given by

$$r_i^I = (w_i - W_i) \times (r_S - R_S) \quad (9)$$

Lastly, pricing returns r_i^P are given by

$$r_i^P = w_i \times (r_i - R_i) \quad (10)$$

The sum of the four sources of return equals the overall active return of the portfolio against its benchmark:

$$r_S^{AA} + r_S^{SS} + r_S^I + r_S^P = r - R \quad (11)$$

Notes

- FIA supports multiple levels of asset allocation decision. In this case, there are as many asset allocation terms as there are sectors used.
- The presence of an interaction term is a shortcoming in the Brinson model. It is often combined with stock selection.
- In the Brinson model, asset allocation is only shown at the sector level, while all other sources of return are shown at the security level.
- In the case that $r_i = R_i$, then pricing return for security i is zero.
- In the case that a security is present in the portfolio but not in the benchmark, or vice versa, then pricing return for that security is zero.

Normalised stock selection

Using the same notation as above, the normalised stock selection approach calculates stock selection as follows:

$$\tilde{w}_i = w_i / w_S \quad (12)$$

$$\tilde{W}_i = W_i / W_S \quad (13)$$

Stock selection return is given by

$$r_i^{SS} = w_S^P (\tilde{w}_i - \tilde{W}_i) (R_i - R_S) \quad (14)$$

Notes

- In this model, returns due to asset allocation and price are the same as for the Brinson approach.
- Interaction return does not arise using this model.

Bottom-up attribution

Here, asset allocation return is given by

$$r_S^{AA} = (w_i - W_i) \times (R_i - R) \quad (15)$$

and stock selection by

$$r_i^{SS} = w_i \times (r_i - R_i) \quad (16)$$

Notes

- Bottom-up attribution is the only model in which it makes sense to show asset allocation returns at the security level.
- Stock selection in the bottom-up model is the same as price return in the previous two models.
- Interaction return does not arise using this model.

Worked examples

The reader is referred to the accompanying spreadsheet 'Equity attribution using Brinson, normalised and bottom-up approaches' for a worked example of all returns shown in this paper.

References

'Equity attribution using Brinson, normalised and bottom-up approaches', Excel spreadsheet, Flametree, dated 15th October 2021